Comparison of Sleep Models for Score-Fatigue Model Integration

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Abstract

The objective of this work was to validate three sleep prediction models using empirical data from the open source literature. The following sleep models were implemented and verified: Two-Process Model of Sleep Regulation by Acherman and Borbley, Three-Process Model of Alertness by Akersted's group, and Sleep/Wake Behavior Model by Darwent's group. The sleep models were extracted from the main program and tested against normal conditions and shift work schedules. The most appropriate model was selected based on generated results.

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1 Introduction

1.1 Objective

The objective of this work was to validate three sleep models versus the empirical data provided in Reference [4]. The following models were programmed and verified: Two-Process Model of Sleep Regulation by Acherman and Borbley [1-2], Three-Process Model of Alertness by Akersted's group [3-5], and Sleep/Wake Model by Darwent's group [6-7].

1.2 Scope

This work included the following tasks:

1. Implement and confirm the programmed algorithms versus the mathematical models provided in References [1] to [7];

2. Validate the models' sleep prediction versus the ideal scenario (normal sleep), when sleep/wake times were not restricted by scheduled duty period;

3. Compare the output results of sleep models to the empirical data provided in Reference [4];

4. Check and assess the implementation of the equations in the main program.

These tasks were completed by the project team; consisting of members from both DRDC and industry.

Project Team Members

Ms. Natalia Doubova	Contractor
Dr. Henry Peng	Co-Lead - DRDC Toronto Research Centre
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This report has the following sections: Introduction – this Section, Method – Section 2, Results – Section 3, and Conclusion – Section 4.

2 Method

2.1 Model 1 – Two-Process Model of Sleep Regulation by Acherman and Borbley [1-2]

The model includes two main components [1-2]: a circadian system and a sleep homeostatic system. The homeostatic sleep pressure as a function of time is as follows:

$$\boldsymbol{S_t = 1 - e^{\frac{-\Delta t}{\tau_r}} * (1 - S_{t-\Delta t})} \quad \textit{during wake} \qquad (1)$$

$$\boldsymbol{S_t = e^{\frac{-\Delta t}{\tau_d}} * S_{t-\Delta t}} \quad \textit{during sleep} \qquad (2)$$

Where Δt is the time step; and τ_r (= 18.2 h) and τ_d (= 4.2 h) are time constants for the rise and decay of the homeostatic process during wakefulness and sleep, respectively.

The circadian rhythm as a function of time is as follows:

$$C(t) = A\sum_{k=1}^{5} a_k \sin\frac{2k\pi}{\tau}(t - t_0) \qquad (3)$$

Where

- A = amplitude of skewed sine wave (= 0.12)
- τ = period of C (24 hours);
- t_0 = the circadian phase at the beginning of the simulation (= 8.6 h)
- a_1 = 0.97
- a_2 = 0.22
- a_3 = 0.07
- a_4 = 0.03
- a_5 = 0.001

The following initial conditions are used in the simulation: at t = 2300 h, S = 0.62166

If **S > 0.67+C** during rest period, it is time to sleep and if **S < 0.17+C**, it is time to wake up.

2.2 Model 2 – Three-Process Model (TPM) of Alertness by Akersted's group [3-5]

TPM includes a *Process C* that represents sleepiness due to circadian influences and has a sinusoidal form with an afternoon peak and a *Process S* that is an exponential function of the time since awakening. At sleep onset, *process S* is reversed and called *S'*. It has been assumed that *C' = C* during sleep, Reference [4]. *Process U* that represents a 12-h period is associated with the afternoon reduction in alertness. The equations describing the TPM are shown below:

$$\boldsymbol{S = la + (sw - la) * e^{(d*ta_w)}} \qquad (4)$$

$$S' = ha - (ha - ss) * e^{(g*ta_s)} \tag{5}$$

$$C = m_1 + \alpha_1 * cos\left(\frac{\pi}{12} * (t - p)\right) \tag{6}$$

$$U = m_2 + \alpha_2 * cos(\frac{\pi}{6} * (t - (p + 3))) \tag{7}$$

Where

la	= low asymptote (= 2.4);
sw	= S at waking up;
d	= decay in alertness (= - 0.0353);
ta_w	= time since waking up, in decimal hours;
ta_s	= time since falling asleep, in decimal hours;
ha	= high asymptote (= 14.3);
ss	= S at falling asleep;
g	= decay which is calculated as $\ln(\frac{ha-14.0}{ha-7.96})/8 \approx -0.381$;
m_1	= mesor (0);
m_2	= mesor (-0.5);
α_1	= amplitude (= 2.5);
α_2	= amplitude (= 0.5);
p	= acrophase in decimal hours (16.48 [3]);
t	= time of day, in decimal hours

Initial conditions:

t = 23 h;
S'(23) = 7.96;

In the present case (Reference [4]), we assume that bedtime in connection with day work occurs at a level **S+C = 8.5** which corresponds to a normative evening bedtime at 23:00. If during sleep, **S+C > 14.2** then it is time to wake up.

In order to obtain sleepiness, the Karolinska Sleepiness Scale (KSS) was applied using the following equation.

$$KSS = a - (b * al) \tag{8}$$

Where

a	= 10.3 ;
b	= 0.47;
al	= S+C for Model 2 Versions 1.1 and 1.2; S+C+U for the other versions

Based on References [3-5], five versions of work/sleep model were developed. These versions are briefly described below.

2.2.1 Model 2 Version 1

Version 1 includes the following equations: homeostatic component during waking S (Equation 4), homeostatic component during sleep *S'* (Equation 5) and circadian component *C* (Equation 6). This version utilizes Version 1.1 with the amplitude (α_1 = 2.5) and Version 1.2 with two circadian rhythms of

different amplitudes (α_1 = 2.5 when awake and α_1 = 0.6 when asleep). The threshold equation is "S+C" for Version 1, Reference [4].

2.2.2 Model 2 Version 2

Version 2 includes the following equations: homeostatic component during awake S (Equation 4), homeostatic component during sleep *S'* (Equation 5), circadian component *C* (Equation 6), and ultradian component *U* (Equation 7). The additional equation is added to the threshold "S+C+U" as described in Reference [4].

2.2.3 Model 2 Version 3

Model 2 Version 3 is not presented in this report because the "break" point used in this version was changed based on Reference [4].

2.2.4 Model 2 Version 4 [4]

Version 4 includes Equation 4, 5, 6, and 7 and the threshold equation "S+C+U". The original process S' (Equation 5) was modified with a "break function" that splits the process into *S'b1* and *S'b2*. The "break point" during sleep is as follows:

$$\boldsymbol{bt = (bl - ss)/(g * (bl - ha))} \qquad (9)$$

Where

bl = break level (= 12.2);
dt = delta time since falling asleep in decimal hours;
ss = S at falling asleep.

If $dt \leq bt$, the process *S'b1* is as follows:

$$\boldsymbol{S'b1 = ss + dt * (g * (bl - ha))} \qquad (10)$$

In the case $dt \geq bt$, the following equation was used:

$$\boldsymbol{S'b2 = ha - (ha - bl) * e^{(g*(\mathrm{ta}_s - bt))}} \qquad (11)$$

2.2.5 Model 2 Version 5 [5]

Version 5 includes the same equations as Version 4; however, some parameters were updated based on Reference [5]. The updated parameters are shown below:

p = 16.8
$S+C+U$ < 8.38 – threshold for falling asleep
$S+C+U$ > 11.38 – threshold for waking up
a = 10.6 (Equation 8)
b = 0.6 (Equation 8)

2.3 Model 3 by Darwent's group

2.3.1 Model 3 Version 1 [6]

This model includes (i) sleep propensity and (ii) a sleep onset/offset threshold rhythm. The equation used to calculate the probabilities makes the assumption of ideal sleep, with no sleep deprivation or sleep restrictions put in place. The derived equation to model these probabilities is as follows:

$$\boldsymbol{f(t) = 0.5 + 0.5 * cos\left(\frac{2\pi t}{24+P}\right)} \qquad (12)$$

Where

f(t)	= sleep propensity at t
t	= time of day
P	= phase offset

Equation 12 sets the probability of being asleep at any given time with the reference to a threshold value. The threshold value is established by the equation below:

$$\boldsymbol{s = \frac{\sum_{i=1}^{12} f(t-i) + \sum_{i=1}^{12} f(t+i)}{n}} \qquad (13)$$

This equation utilizes the pre-calculated probability quotients from the baseline regression equation (Equation 13) in the 12 hours before and 12 hours after time, t. The timing and duration of sleep periods are estimated by comparing the sleep propensity and the sleep onset/offset threshold rhythm. Sleep propensity during scheduled work periods is assumed to be zero [6]. Whenever the threshold value is greater than the predicted sleep probability for a particular time then the person is considered to be awake, whereas during the points when the threshold value is less than the predicted sleep probability the subject is considered asleep.

2.3.2 Model 3 Version 2 [7]

Version 2 was developed based on Reference [7]. The initial objective was to define a regression equation to model sleep probability under normal, or baseline, sleep/wake conditions. A modified form of a standard cosine equation was defined to model ideal sleep, with no sleep deprivation or sleep restrictions put in place. The equation is described below:

$$\boldsymbol{f(t) = \left(M + A * \cos\left(\frac{2\pi t}{24} + P\right)\right)^{S}} \qquad (14)$$

Where

f(t)	= predicted sleep probability quotient at t
t	= time of day
M	= midline-estimating statistics of the rhythm (MESOR) of the cosine function (= 0.5)
A	= amplitude of the cosine function (= 0.5)
P	= phase of cosine function
S	= slope of the distribution

The MESOR, M, and the amplitude, A, were assigned a constant value of 0.5 [7]. The phase offset parameter, P, and the exponential parameters, were determined to be 5.365 and 2.810 respectively.

Equation 14 sets the probability of being asleep at any given time with reference to a threshold value. The algorithm to calculate the threshold value is given by:

$$\boldsymbol{s}_{\mathbf{on-off}} = \frac{\sum_{i=1}^{12} \boldsymbol{f}(\boldsymbol{t}-\boldsymbol{i}) + \sum_{i-1}^{12} \boldsymbol{f}(\boldsymbol{t}+\boldsymbol{i})}{\sum_{i=0}^{24} \boldsymbol{f}(\boldsymbol{i}) \times \boldsymbol{S}} \qquad (15)$$

Where

s = derived variable representing the effect of sleep homeostasis and sleep anticipation on sleep probability

$\sum_{i=1}^{12} f(t-i)$ = Sum of sleep propensity in the 12 hours prior to time t

$\sum_{i=1}^{12} f(t+i)$ = Sum of expected sleep probability quotients in the 12 hours after time t

$\sum_{i=0}^{23} f(t)$ = Sum of predicted sleep probability quotients derived from Equation 14 over the time interval $0 \leq T \leq 23$

S = constant; S represents the parameters estimate obtained for S from Equation 14

During work the threshold value is changed to “0” as no sleep occurs during this period of time.

When the threshold value is greater than the predicted sleep probability for a particular time then the person is considered to be awake, and when the threshold value is less than the predicted sleep probability the person is considered to be asleep.

3 Results

The results are outlined below:

- All three models have been validated versus the ideal scenario of sleep. The results are provided in Table 1, Table 3, Table 4, Table 5, Table 6, Table 9, and Table 11. Model 1 is the most appropriate model under normal conditions;

- The model predictions of sleep using the shift worker schedule are provided in Table 2, Table 7, and Table 10. Model 1, Model 2 Version 5 and Model 3 Version 2 are in a good agreement with the empirical results;

- The mean square error between empirical data and the model prediction for Model 2 are summarized in Table 8.

3.1 Model 1 – Two-Process Model of Sleep Regulation

Sleep/Wake time under normal conditions are provided in **Table 1**.

Table 1: Sleep/Wake Time under Normal Conditions (Model 1)

Acherman and Borbley Model [1-2]			
Sleep start time	**Time**	**Sleep stop time**	**Time**
27/10/2013	2300	28/10/2013	700
28/10/2013	2300	29/10/2013	715
29/10/2013	2315	30/10/2013	715
30/10/2013	2315	31/10/2013	715
31/10/2013	2315	01/11/2013	715

Sleep/work schedule for shift worker is shown in **Table 2**.

Table 2: Sleep/Wake/Work Time for Shift Worker (Model 1)

Acherman and Borbley Model [1-2]							
Sleep start date	**Time**	**Sleep stop date**	**Time**	**Work start date**	**Time**	**Work stop date**	**Time**
2013-10-27	2300	2013-10-28	700	29/10/2013	2100	30/10/2013	600
2013-10-28	2300	2013-10-29	715	30/10/2013	1400	30/10/2013	2100
2013-10-30	600	2013-10-30	1030	31/10/2013	600	31/10/2013	1400
2013-10-30	2300	2013-10-31	600				
2013-10-31	2215	2013-11-01	0				

Model 1 files are located on ..\..\nd\acherman_borely.

3.2 Model 2 – Three-Process Model (TPM)

3.2.1 Model 2 Version 1.1 and Version 1.2

Model 2 Version 1.1 and Version 1.2 were tested under normal conditions. The results are shown in **Table 3**.

Table 3: Sleep/Wake Time under Normal Conditions (Version 1.1 and Version 1.2)

Version 1.1 ($\alpha_1 = 2.5$)			
Sleep start date	**Time**	**Sleep stop date**	**Time**
27/10/2013	2300	28/10/2013	700
28/10/2013	2300	29/10/2013	1030
30/10/2013	30	30/10/2013	1030
31/10/2013	30	31/10/2013	1030
01/11/2013	30	01/11/2013	1030
Version 1.2 ($\alpha_1 = 2.5$ & 0.6)			
Sleep start date	**time**	**Sleep stop date**	**Time**
27/10/2013	2300	28/10/2013	700
28/10/2013	2300	29/10/2013	1030
30/10/2013	30	30/10/2013	1045
31/10/2013	30	31/10/2013	1030
01/11/2013	30	01/11/2013	1030

Version 1.1 and 1.2 files (normal conditions) are located on ..\..\nd\Work_Sleep_model2_V1\Model2V1_testTime and ..\..\nd\Work_Sleep_model2_1_alpha\Model2V1_alphaTestTime

3.2.2 Model 2 Version 2

This model was tested under normal conditions; the results are shown in **Table 4**.

Table 4: Sleep/Wake time under Normal Conditions (Model 2 Version 2)

Version 2			
Sleep start date	**Time**	**Sleep stop date**	**Time**
27/10/2013	2300	28/10/2013	700
28/10/2013	2300	29/10/2013	1145
30/10/2013	0	30/10/2013	1145
31/10/2013	0	31/10/2013	1145
01/11/2013	0	01/11/2013	1030

Model 2 Version 2 files (normal conditions) are located on
..\..\nd\Work_Sleep_model2_V2\M2V2_testTime

3.2.3 Model 2 Version 3

There are no results for Model 2 Version 3 because the "break" point used in this version was changed based on Reference [4].

3.2.4 Model 2 Version 4

The results of this version under normal conditions are provided in **Table 5**.

Table 5: Sleep/Wake Time under Normal Conditions (Model 2 Version 4)

Version 4			
Sleep start date	**Time**	**Sleep stop date**	**Time**
27/10/2013	2300	28/10/2013	700
28/10/2013	2300	29/10/2013	1200
30/10/2013	0	30/10/2013	1145
31/10/2013	0	31/10/2013	1145
01/11/2013	0	01/11/2013	1030

Model 2 Version 4 files (normal conditions) are located on
..\..\nd\Work_Sleep_model2_V4\M2V4_testTime

3.2.5 Model 2 Version 5

The results of this version under normal conditions are provided in **Table 6**.

Table 6: Sleep/Wake Time under Normal Conditions (Model 2 Version 5)

Version 5			
Sleep start time	**Time**	**Sleep stop time**	**Time**
27/10/2013	2300	28/10/2013	700
28/10/2013	2300	29/10/2013	700
29/10/2013	2245	30/10/2013	645
30/10/2013	2300	31/10/2013	645
31/10/2013	2300	01/11/2013	645

The model predictions of sleep using the shift worker schedule and all Sleep/Wake/Work times for all versions of Model 2 are provided in **Table 7**.

Model 2 Version 1.1 files are located on ..\..\nd\Work_Sleep_model2_V1
Model 2 Version 1.2 files are located on ..\..\nd\Work_Sleep_model2_1_alpha
Model 2 Version 2 files are located on ..\..\nd\Work_Sleep_model2_V2.
Model 2 Version 4 files are located on ..\..\nd\Work_Sleep_model2_V4.
Model 2 Version 5 files are located on ..\..\nd\Work_Sleep_model2_V5

Table 7: Sleep/Wake/Work Time for Shift Worker (Model 2)

Version 1 (alpha=2.5)							
Sleep start date	**Time**	**Sleep stop date**	**Time**	**Work start date**	**Time**	**Work stop date**	**Time**
2013-10-27	2300	2013-10-28	700	29/10/2013	2100	30/10/2013	600
2013-10-28	2300	2013-10-29	1030	30/10/2013	1400	30/10/2013	2100
2013-10-30	600	2013-10-30	1130	31/10/2013	600	31/10/2013	1400
2013-10-31	15	2013-10-31	600				
2013-10-31	2300	2013-11-01	0				
Version 1 (alpha = 2.5 & 0.6)							
Sleep start date	**Time**	**Sleep stop date**	**Time**	**Work start date**	**Time**	**Work stop date**	**Time**
2013-10-27	2300	2013-10-28	700	29/10/2013	2100	30/10/2013	600
2013-10-28	2300	2013-10-29	1030	30/10/2013	1400	30/10/2013	2100
2013-10-30	600	2013-10-30	1300	31/10/2013	600	31/10/2013	1400
2013-10-31	100	2013-10-31	600				
2013-10-31	2245	2013-11-01	0				
Version 2							
Sleep start date	**Time**	**Sleep stop date**	**Time**	**Work start date**	**Time**	**Work stop date**	**Time**
2013-10-27	2300	2013-10-28	700	29/10/2013	2100	30/10/2013	600
2013-10-28	2300	2013-10-29	1145	30/10/2013	1400	30/10/2013	2100
2013-10-30	600	2013-10-30	1245	31/10/2013	600	31/10/2013	1400
2013-10-31	0	2013-10-31	600				
2013-10-31	2230	2013-11-01	0				
Version 4							
Sleep start date	**Time**	**Sleep stop date**	**Time**	**Work start date**	**Time**	**Work stop date**	**Time**
2013-10-27	2300	2013-10-28	700	29/10/2013	2100	30/10/2013	600
2013-10-28	2300	2013-10-29	1200	30/10/2013	1400	30/10/2013	2100
2013-10-30	600	2013-10-30	1315	31/10/2013	600	31/10/2013	1400
2013-10-30	2345	2013-10-31	600				
2013-10-31	2230	2013-11-01	0				

Version 5							
Sleep start date	**Time**	**Sleep stop date**	**Time**	**Work start date**	**Time**	**Work stop date**	**Time**
2013-10-27	2300	2013-10-28	700	29/10/2013	2100	30/10/2013	600
2013-10-28	2300	2013-10-29	700	30/10/2013	1400	30/10/2013	2100
2013-10-30	600	2013-10-30	1130	31/10/2013	600	31/10/2013	1400
2013-10-30	2245	2013-10-31	600				
2013-10-31	2245	2013-11-01	0				
Empirical Results							
Sleep start date	**Time**	**Sleep stop date**	**Time**	**Work start date**	**Time**	**Work stop date**	**Time**
2013-10-30	622	2013-10-30	1115	29/10/2013	2100	30/10/2013	600
2013-10-30	2235	2013-10-31	445	30/10/2013	1400	30/10/2013	2100
				31/10/2013	600	31/10/2013	1400

3.2.6 Mean Square Error (MSE)

The mean square error for each version of Model 2 is shown in **Table 8**.

Table 8: Mean Square Error for Model 2

Version 1 (α_1 = 2.5)	
Mean Square Error:	0.55279
Version 1 (α_1 = 2.5 & 0.6)	
Mean Square Error:	0.72307
Version 2	
Mean Square Error:	0.62366
Version 4	
Mean Square Error:	0.59931
Version 5	
Mean Square Error:	1.3155

Figure 1 illustrates the empirical and predicted sleepiness in the rapidly rotating shift schedule. The parameters in Equation 8 were changed in Version 5.

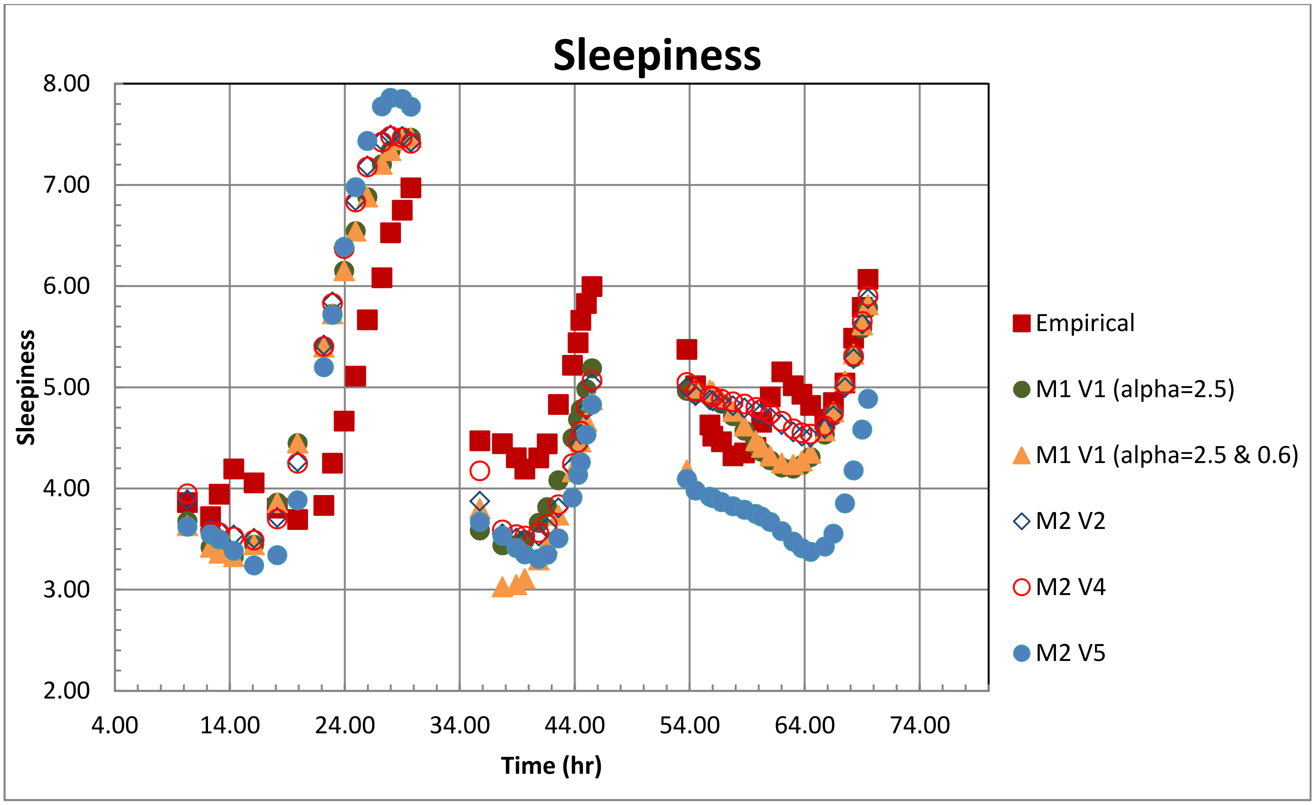

Figure 1: Predicted and Empirical Sleepiness from [4] using Model 2

3.3 Model 3

3.3.1 Model 3 Version 1

This model was validated using normal conditions. The results are shown in **Table 9**.

Table 9: Sleep/Wake Time under Normal Conditions (Model 3 Version 1)

Model 3 Version 1			
Sleep start time	**Time**	**Sleep stop time**	**Time**
28/10/2013	0	28/10/2013	800
29/10/2013	0	29/10/2013	730
29/10/2013	2215	30/10/2013	1245
31/10/2013	330	31/10/2013	1815
01/11/2013	0	01/11/2013	15

Cosine function (Equation 12) does not have a period of 24 hours; as a result, the predicted sleep and wake time are not consistent under normal conditions. This model needs to be revised.

Model 3 Version 1 files are located on ..\..\nd\Work_Sleep_model3_V1

3.3.2 Model 3 Version 2

Sleep/work schedule under different values of s and f(t) during work are tabulated in **Table 10**. It has been observed that 15 minutes asleep is required after work if s_{on-off}=0 during work and f(t)≠0.

Table 10: Sleep/Wake/Work Time for Shift Worker (Model 3 Version 2)

Model 3 Version 2 s_{on-off}(working)=0, f(working)<>0							
Sleep start date	**Time**	**Sleep stop date**	**Time**	**Work start date**	**Time**	**Work stop date**	**Time**
2013-10-28	0	2013-10-28	800	29/10/2013	2100	30/10/2013	600
2013-10-29	0	2013-10-29	815	30/10/2013	1400	30/10/2013	2100
2013-10-30	600	2013-10-30	1015	31/10/2013	600	31/10/2013	1400
2013-10-30	2100	2013-10-30	2115				
2013-10-30	2300	2013-10-31	600				
2013-10-31	1400	2013-10-31	1415				
2013-10-31	2300	2013-11-01	0				
Model 3 Version 2 s_{on-off}(working)=0, f(working)=0 (sleep conditions are changed to f(t)>S!!!)							
Sleep start date	**Time**	**Sleep stop date**	**Time**	**Work start date**	**Time**	**Work stop date**	**Time**
2013-10-28	0	2013-10-28	800	29/10/2013	2100	30/10/2013	600
2013-10-29	0	2013-10-29	815	30/10/2013	1400	30/10/2013	2100
2013-10-30	615	2013-10-30	1015	31/10/2013	600	31/10/2013	1400
2013-10-30	2300	2013-10-31	600				
2013-10-31	2300	2013-11-01	0				
Model 3 Version 2 s_{on-off}(working)<>0, f(working)=0							
Sleep start date	**Time**	**Sleep stop date**	**Time**	**Work start date**	**Time**	**Work stop date**	**Time**
2013-10-28	0	2013-10-28	800	29/10/2013	2100	30/10/2013	600
2013-10-29	0	2013-10-29	815	30/10/2013	1400	30/10/2013	2100
2013-10-30	615	2013-10-30	1015	31/10/2013	600	31/10/2013	1400
2013-10-30	2300	2013-10-31	600				
2013-10-31	2300	2013-11-01	0				

Model 3 Version 2 files are located on ..\..\nd\Work_Sleep_model3_V2.

Model 3 Version 2 was tested under normal conditions. The results are shown in Table 11.

Table 11: Sleep/Wake Time under Normal Conditions (Model 3 Version 2)

Model 3 Version 2			
Sleep start time	**Time**	**Sleep stop time**	**Time**
28/10/2013	0	28/10/2013	800
29/10/2013	0	29/10/2013	815
29/10/2013	2300	30/10/2013	815
30/10/2013	2300	31/10/2013	815
31/10/2013	2300	01/11/2013	815

The following findings can be reported:

- Initial time for the Circadian function of Model 1 in the main program was defined as t_0=0.49 instead of time used as the initial conditions;
- Model 1: An extra time step was added when C(t) was calculated. As a result, C(t) is one delta time ahead: $C_1(t_0+\Delta t)$, $C_2(t_0+2\Delta t)$, $C_3(t_0+3\Delta t)$ etc. The correct circadian function has to be calculated as follows: $C_1(t_0)$, $C_2(t_0+\Delta t)$; $C_3(t_0+3\Delta t)$ etc where t_0 = 23 pm.
- The number of columns for each position in the csv files was calculated based on nonempty cells in the second line. If any other position has a different number on nonempty cells, the result will be incorrect.
- 15 minutes of sleep was added to the sleep schedule in the original m-file.
- AddBaseLineDatesandTimes function: after the baseline days were added, additional 30 minutes were added to start computation; for example, instead of 12 am the computation will start at 00:30.
- An extra record was created in the work schedule with start time of 0 and end time of 0.

4 Conclusion

Based on the results provided in Section 3, the following conclusion can be drawn:

- The most appropriate sleep model to implement is the Model 1;
- All reported bugs in programming have been fixed;
- Model 1 and Model 2 Version 5 have been tested using the main program.

The following is recommended for the next step:

- Perform further validation of sleep models versus the additional empirical data;
- Add sleep inertia to Model 2 Version 5;
- Test Model 3 using the main program.

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List of symbols/abbreviations/acronyms/initialisms

DRDC	Defence Research & Development Canada
C	Circadian Rythm
KSS	Karolinska Sleepiness Scale
MSE	Mean Square Error
S_t	Homeostatic sleep pressure
TPM	Three-Process Model
U	Ultradian component